Measuring Social Progress*

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Abstract

In this chapter, we provide a critical review of the literature on social progress from a prioritarian perspective. We give special attention to the choice of the measure of individual well-being and the aggregation method across individuals in our discussion of various methods that have been proposed in the literature. We present the “growth incidence curve” as a useful device to chart social progress. We illustrate with empirical results for Russia in the period 1995-2005.

Keywords: Social progress, social condition, prioritarian social evaluations, preferences, growth incidence curve.

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1 Introduction

Are the individuals in society \( x \) better off than the individuals in society \( y \)? How much social progress is society \( x \) making over time? The standard (and still most popular) approach to address these questions is to look at GDP per capita. The society with the highest GDP per capita is considered as better off and social progress is measured by the growth of GDP per capita. Yet, while GDP per capita is arguably a useful measure of the size of economic activity, it is by now generally accepted that it is not a good measure of overall social welfare. We see at least two reasons. First, GDP per capita remains blind for non-monetary aspects of well-being and, second, the distribution of well-being across the individuals is left out of the picture. Various proposals have therefore been made to go “Beyond GDP” (see Stiglitz et al. (2009) for a survey).

In this chapter, we critically survey the academic literature that deals with the question how social conditions can be assessed and compared through a multidimensional and prioritarian (or, better, generalized utilitarian) lens. To avoid a sprawling discussion of this vast and rapidly expanding field\(^1\), we focus here primarily on the topic of the measurement of social progress, i.e., the question of how much the well-being of the individuals in a particular society has improved over time. Yet, most of the surveyed results and tools can be applied to the broader topic of the general assessment and comparison of social conditions.

Broadly speaking, measuring and comparing social progress entails two challenges which echo the critiques raised against GDP per capita. First, including non-monetary information in a measure of social progress requires an operational measure of individual well-being. We group the different approaches in this survey according to the underlying definition of individual well-being. We make a distinction between approaches based on objective, subjective (or hedonic) and preferentialist well-being measures (see Chapter 3 of this book). Second, information needs to be included about where progress takes place in the distribution of well-being. Has the well-being of the worst-off improved or is well-being growth rather benefiting individuals in more favourable positions? A central role in our chapter is occupied by the growth incidence curve that was proposed by Ravallion and Chen (2003). This curve is a useful device to chart and visualize well-being growth across the entire distribution. We relate the growth incidence curve to the literature on prioritarian social welfare functions developed in the wake of the work by Atkinson (1970) and Kolm-Pollak (Pollak (1971); Kolm (1976)).

The chapter is organized as follows. In Section 2 we sketch a general theoretical framework to measure social progress in a prioritarian setting. We present two decompositions that

provide additional insights into the structure of the prioritarian considerations in any given context and we discuss how to introduce lifetime well-being and population issues into the analysis. In the third section, we take a critical look at the existing (applied) literature on social progress. Section 4 illustrates the different theoretical concepts introduced in the previous sections with an empirical application based on Russian data from the RMLS-HSE between 1995 and 2005. During this turbulent decade, the Russian economy underwent sharp changes, including the financial crisis of August 1998. We conclude in Section 5.

2 Social welfare and social progress

2.1 Social welfare functions

Let there be \( N \) individuals in society. We start by assuming that \( N \) is constant over time. We discuss and relax that assumption in Section 2.4.2. The well-being situation of individual \( i \) in period \( t \) is described by the bundle of attributes \( a_{it} \). Examples of attributes are income (or consumption), health, job satisfaction, quality of social relations, etc. Since income plays a crucial role in many approaches, we introduce it explicitly in our notation when that is useful by splitting the attribute vector \( a_{it} \) in two parts \( (y_{it}, x_{it}) \), in which \( y_{it} \) represents the income attribute and the vector \( x_{it} \) the non-income attributes.

Define \( a_t = (a_{1t}, \ldots, a_{Nt}) \) as the attribute matrix in period \( t \), which contains for each individual \( i \) her attribute vector \( a_{it} \). We assume that social welfare is given by an instantaneous social welfare function \( S(\cdot) \) that only takes the attribute matrix \( a_t \) of period \( t \) as argument (we return to that assumption in Section 2.4.1. Furthermore, we restrict our attention to the generalized utilitarian social welfare functions:

\[
S(a_t) = \frac{1}{N} \sum_{i=1}^{N} g(w_i(a_{it})),
\]  

where \( g(\cdot) \) is an increasing transformation function. Eq. (1) captures an explicit two-step approach where in a first step a well-being measure \( w_{it} = w_i(a_{it}) \) is computed for each individual \( i \). These \( N \) well-being measures form a vector \( w_t = (w_{1t}, \ldots, w_{Nt}) \). In a second step, the well-being measures are transformed by the function \( g(\cdot) \) and averaged. When \( g(\cdot) \) is the identity function, we call \( S(\cdot) \) a utilitarian social welfare function. When \( g(\cdot) \) is a strictly increasing and strictly concave function, we call \( S(\cdot) \) a prioritarian social welfare function.

In this section, we follow this two-step approach without specifying the precise content of individual well-being. In the next section we briefly discuss three different interpretations of
the well-being measures (i.e., objective, subjective and preferentialist), see also Chapter 3 of this book.

As soon as we have specified a social welfare function, it is natural to assess society’s condition on the basis of that function. A society with attribute matrix \( a_t \) is “better off”, i.e., reaches a higher level of social welfare, than a society with attribute matrix \( a'_t \), if \( S(a_t) > S(a'_t) \). In the same vein, we say that there is social progress if the level of social welfare increases over time:

\[
\Delta S = S(a_{t+1}) - S(a_t) > 0. \tag{2}
\]

In the generalized utilitarian case\(^2\) of Eq. (1), we say that there is social progress if:

\[
\Delta S = \frac{1}{N} \sum_{i=1}^{N} \left[ g(w_i(a_{i(t+1)})) - g(w_i(a_{it})) \right] > 0. \tag{3}
\]

Note that the social welfare function (1) and the measures of social progress according to Eqs. (2) and (3) satisfy an anonymity principle with respect to well-being in each period. Reshuffling the names of the individuals in the attribute matrix of one period does not affect the measurement of social progress. This has the advantage that only cross-sectional data are needed. In Section 2.4.1 we discuss how life-cycle considerations can be introduced in a cross-sectional setting.

If we want to compare the social welfare levels of two societies, or check whether there is social progress or regress, i.e., whether the Eqs. (2)-(3) are positive or negative, it is sufficient that social welfare is measured at an ordinal level. To interpret the numerical value of \( \Delta S \) as the increase in social welfare, however, social welfare must be measured on a translation-scale that is defined up to a constant term. Yet, it is common to measure social progress in terms of growth rates. In the current setting, this means calculating the growth rate of social welfare \( \Delta S/S(a_t) \). This growth rate can only be meaningfully interpreted if it can be assumed that social welfare is measured on a ratio-scale that is defined up to a constant factor. Note that we are not talking here about the measurability level of the individual well-being functions \( w(.) \), but about the measurability level of the function \( S(.) \). Both aspects are closely linked if we choose for the social welfare function a representation in terms of its equally-distributed equivalent.

The equally-distributed equivalent is the well-being level which, if it were equally enjoyed by all individuals, would yield the same level of social welfare as the current distribution of

\(^2\)Bossert and Dutta (2019) discuss the measurement of welfare change using generalized Gini social welfare functions.
well-being. In the generalized utilitarian case, it is given by:

\[
\tilde{S}(a_t) = g^{-1}\left[\frac{1}{N} \sum_{i=1}^{N} g(w_i(a_{it}))\right].
\] (4)

In the following, we use the notation \(\tilde{S}\) to refer to the equally-distributed equivalent representation of the social welfare function. One of the advantages to work with the equally-distributed equivalent representation is that it is expressed in the same units as individual well-being and, hence, more easily interpreted. When all individuals have the same well-being level \(\bar{w}\), the equally-distributed equivalent is also equal to \(\bar{w}\) (see Eq. (4)). When there is some inequality in the well-being distribution, the equally-distributed equivalent will be strictly lower than the average well-being level in the prioritarian case when the transformation function \(g(\cdot)\) is strictly increasing and strictly concave.\(^3\)

If we are willing to assume that individual well-being is perfectly measurable (i.e. both ratio-scale and translation-scale measurable), the equally-distributed equivalent representation of the social welfare function is also perfectly measurable and, hence, absolute changes and growth rates of social welfare can be computed. Perfect measurability is an extremely demanding condition, however. We illustrate less demanding approaches for the two most important subfamilies of prioritarian social welfare functions, the Atkinson and Kolm-Pollak social welfare functions (see Chapter 2 of this book for a more detailed discussion). In the recent literature, the Atkinson specification has been more popular than the Kolm-Pollak specification. In our own empirical application in Section 4, we show the results for both subfamilies.

**The Atkinson social welfare function**

To obtain the Atkinson social welfare function, the transformation function \(g\) is chosen as follows:

\[
g^{ATK}(w_{it}) = \frac{w_{it}^{1-\gamma}}{1 - \gamma},
\] (5)

with \(\gamma > 0\) being a parameter of inequality aversion. The larger \(\gamma\), the larger the degree of priority given to the worse off. In the case when \(\gamma = 1\), \(g^{ATK}(\cdot)\) is the logarithmic transformation function. The equally-distributed equivalent of the Atkinson social welfare function is given by:

\[^3\text{The so-called normative inequality measures are derived from the discrepancy between the level of the equally-distributed equivalent and the average of a distribution. They measure the welfare loss due to inequality. Some examples are given in the next section.}\]
\[
\bar{S}_\gamma^{\text{ATK}}(a_t) = \left[ \frac{1}{N} \sum_{i=1}^{N} (w_i(a_t))^{1-\gamma} \right]^{1/(1-\gamma)}.
\]

If the individual well-being measure is ratio-scale measurable - a necessary condition for the Atkinson social welfare function to make sense - then the growth rate in \(\bar{S}_\gamma^{\text{ATK}}(a_t)\) is also meaningfully defined.

**The Kolm-Pollak social welfare function**

We obtain the Kolm-Pollak social welfare function by selecting the following transformation function \(g\):

\[
g^{KP}(w_{it}) = -\frac{\exp(-\beta w_{it})}{\beta},
\]

with \(\beta > 0\) a parameter of inequality aversion. The equally-distributed equivalent of the Kolm-Pollak social welfare function can be written as follows:

\[
\bar{S}_\beta^{KP}(a_t) = -\frac{1}{\beta} \log \left[ \frac{1}{N} \sum_{i=1}^{N} \exp\left(-\beta w_i(a_t)\right) \right].
\]

If the well-being measure is translation-scale measurable, we can meaningfully measure the value of \(\Delta \bar{S}_\beta^{KP}(a_t)\), i.e., the absolute change in social welfare.

### 2.2 Decomposing social progress

The crucial difference between the utilitarian and the prioritarian approach to the measurement of social welfare is that in the latter approach a redistribution from the better off to the worse off is seen as a social improvement due to the strict concavity of the transformation function \(g(.)\) in Eq. (1). The degree of concavity determines the relative weight given to the worse off. The importance of prioritarian considerations can be made more salient by two decompositions. First, we discuss a decomposition of the level of social welfare and, second, we present a decomposition of the change in social welfare.

#### 2.2.1 Mean and inequality

A first decomposition is to write the equally-distributed equivalent representation of social welfare as a function of the average and the inequality of individual well-being. As discussed in Chapter 2 of this book, it is convenient to use a different decomposition for the Atkinson
and the Kolm-Pollak social welfare functions, so that a relative inequality index is obtained in the former case and an absolute inequality index in the latter.\footnote{A relative inequality index is invariant to a rescaling of all values with a common factor and an absolute inequality index is invariant to a translation of all values with a common constant.}

For the Atkinson specification, we can write:

$$\tilde{S}_{\gamma}^{ATK}(a_t) = \mu(a_t)[1 - A_{\gamma}(a_t)], \quad (9)$$

with $\mu(a_t) = \sum_i (w_i(a_{it})) / N$ being the average well-being in period $t$ and $A_{\gamma}(a_t)$ the Atkinson-index of inequality in well-being for the inequality aversion parameter $\gamma$. In Eq. (9), the inequality index “corrects” the average well-being to take the loss in social welfare due to well-being inequality into account. The larger well-being inequality, the larger this correction becomes. In the utilitarian case ($\gamma = 0$) we have that $A_{\gamma}(a_t) = 0$ for any attribute matrix $a_t$, and social welfare equals average well-being. In this case, social progress equals the change in average well-being.

A similar decomposition is possible for the Kolm-Pollak specification, for which:

$$\tilde{S}_{\beta}^{KP}(a_t) = \mu(a_t) - K_{\beta}(a_t), \quad (10)$$

with $K_{\beta}(a_t)$ the Kolm-Pollak measure of inequality for the inequality aversion parameter $\beta$. Again, for the utilitarian social welfare function ($\beta = 0$), the inequality measure equals 0 and in this case social welfare equals average well-being, so that social progress is measured by the change in average well-being.

### 2.2.2 Visualizing well-being changes: growth incidence curves

The second decomposition deals with the change in social welfare over time and is based on the concept of a growth incidence curve. For this decomposition it is useful to distinguish between the relative approach in terms of percentage-wise growth rates (using the Atkinson social welfare function) and an absolute approach in terms of social welfare differences (using the Kolm-Pollak social welfare function). We discuss both approaches here and illustrate them with Russian data in Section 5. In this section, we switch to continuous notation because this is more convenient to introduce the decompositions.

Let $Q_t$ be the quantile function of the well-being distribution $w_t$, i.e., the function that returns the well-being level at time $t$ for each percentile $p$.\footnote{If $F_t$ is the cumulative distribution function of the individual well-being measures in period $t$, we have that for each percentile $p$, $Q_t(p) = w_i(a_{it})$ if and only if $p = F_t(w_i(a_{it}))$.} In this continuous framework,
we can restate Eq. (4) as:

$$\tilde{S}_t = \tilde{S}(a_t) = g^{-1}\left[\int_0^1 g(Q_t(p)) \, dp \right].$$  \hspace{1cm} (11)

**The relative growth incidence curve and the Atkinson social welfare function**

Ravallion and Chen (2003) have introduced the relative growth incidence curve, which we denote $rGIC_t(p)$. This curve shows the (relative) well-being growth rate at percentile $p$:

$$rGIC_t(p) = \frac{dQ_t(p)/Q_t(p)}{dt}. \hspace{1cm} (12)$$

Interestingly, there is a direct link between this relative growth incidence curve and the relative growth rate of (the equally-distributed equivalent of) the Atkinson social welfare function. As shown in the Appendix, we can decompose the relative growth rate of the Atkinson social welfare function as follows:

$$\frac{d\tilde{S}^{ATK}_t/\tilde{S}^{ATK}_t}{dt} = \int_0^1 \left[ \frac{g^{ATK}(Q(p))}{\int_0^1 g^{ATK}(Q(q)) \, dq} \right] \times rGIC_t(p) \times dp. \hspace{1cm} (13)$$

This means that the growth rate of the Atkinson social welfare can be written as a weighted sum of the growth rates of well-being at the different percentiles as given by the relative growth incidence curve. The weights, which are indicated between square brackets in Eq. (13), are given by the ratio of the transformed well-being at each percentile and the average transformed well-being across all percentiles.

In the utilitarian case ($\gamma = 0$), the weights are equal to the ratio of well-being at each percentile and average well-being. Perhaps counter-intuitively, higher percentiles then get a larger weight. In the prioritarian case ($\gamma > 0$), however, the higher the parameter $\gamma$, the larger the weight that is given to the growth rates of the worse-off percentiles.

**The absolute growth incidence curve and the Kolm-Pollak social welfare function**

The absolute growth incidence curve, denoted $aGIC_t(p)$, shows the (absolute) growth of well-being at percentile $p$ of the well-being distribution:

$$aGIC_t(p) = \frac{dQ_t(p)}{dt}. \hspace{1cm} (14)$$

Comparing Eq. (12) and (14), we see that $rGIC_t(p) = aGIC_t(p)/Q_t(p)$.

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6See also Kraay (2006) and Ferreira (2012) for similar discussions.
In the case of the Kolm-Pollak social welfare function, there is an analogous link between the absolute growth incidence curve and the absolute change in social welfare. As shown in Appendix, we obtain:

\[
\frac{d\tilde{S}_t^{KP}}{dt} = \int_0^1 \left[ \frac{g^{KP}(Q(p))}{\int_0^1 g^{KP}(Q(q))dq} \right] \times aGIC_t(p) \times dp,
\]  

(15)

The absolute growth of the Kolm-Pollak social welfare function can be written as a weighted sum of the growth of well-being at the different percentiles, as given by the absolute growth incidence curve. Similarly to the Atkinson social welfare function, the weights assigned to the absolute growth incidence curve at each percentile are given by the ratio of the transformed well-being at each percentile and the average transformed well-being across all percentiles.

In the utilitarian case \((\beta = 0)\), all weights become equal to 1 and the growth of social welfare is equal to the average change in individual well-being. In the prioritarian case \((\beta > 0)\), the larger the parameter \(\beta\) is, the larger the weight that is given to the well-being growth of the worse off. Contrary to the decomposition of the percentage-wise change in social welfare in the Atkinson case, the weights in the decomposition of the change in the Kolm-Pollak social welfare function are decreasing with \(p\).

### 2.3 Inequality aversion and the well-being measure

Eq. (1) allows us to focus on two central choices when measuring social progress, i.e., the choice of the social welfare function (see chapter 2, this volume) and the choice of the individual well-being (or utility) function (see chapter 3, this volume). These two choices can be made independently of each other. However, in much of the applied work, the two choices are intertwined so that it is impossible to identify (the curvature of) both functions separately.

Let us illustrate the issue for the simple case in which social welfare is measured based on information about the income distribution \(y_t = (y_{1t}, \ldots, y_{Nt})\). It is common to work with the functional form:

\[
S(y_t) = \frac{1}{N} \sum_{i=1}^{N} h(y_{it}).
\]  

(16)

From the perspective of Eq. (1), it is tempting to interpret Eq. (16) as utilitarian. However, this is not the common interpretation in the literature, which interprets \(h(.)\) as the concatenation of \(g(.)\) and \(w(.)\), i.e., \(h = g \circ w\). This means that “inequality aversion” (of the social welfare function) and “risk aversion” (of the utility function) can no longer be distinguished,
if one wishes to do so. This can be easily illustrated. Suppose one takes a utility function:

\[ w(y_{it}) = \frac{y_{it}^{1-\lambda}}{1 - \lambda}, \quad (17) \]

with \( \lambda \) representing relative risk aversion, and the Atkinson social welfare function

\[ S(y_t) = \frac{1}{1 - \gamma} \frac{1}{N} \sum_{i=1}^{N} w(y_{it})^{1-\gamma}, \quad (18) \]

with \( \gamma \) the parameter of inequality aversion. Combining equations (17) and (18) yields

\[ S(y_t) = \frac{1}{(1 - \gamma)(1 - \lambda)} \frac{1}{N} \sum_{i=1}^{N} y_{it}^{(1-\gamma)(1-\lambda)}, \quad (19) \]

and this can also be interpreted as a specification of a linear utility function and inequality aversion equal to \( 1 - (1 - \gamma)(1 - \lambda) \). The interpretation goes both ways: it would be wrong to say that all applied workers that implement Eq. (19) are assuming a linear (risk-neutral) utility function. The specification (16) has become popular, precisely because it avoids the difficult issue of separating risk aversion and inequality aversion and allows to focus directly on the ethical issues raised by the unequal distribution of incomes.

2.4 Issues of life and death: lifetime well-being and variable population

The prioritarian approach to social welfare naturally starts from a lifetime perspective on individual well-being. If we want to determine which individuals are better off, it is indeed necessary to consider their entire life course. Most applied work, however, focuses on social welfare in one period (usually a year). Introducing life-cycle considerations into an empirical analysis raises a host of tricky issues, both from a conceptual (see, e.g., Adler (2012), Ponthière (2016)) and a practical viewpoint. In Section 2.4.1, we focus on the latter. In Section 2.4.2, we briefly discuss the related issue of how to handle variable populations.

2.4.1 Lifetime well-being

At each moment in time we have a situation with overlapping generations, i.e., with persons of different generations living together. To illustrate its implications, we have to extend the framework of the previous section by including more periods. Let us consider a simple example with three individuals \( i, j, \) and \( k \) and three periods \( (t - 1), t \) and \( (t + 1) \). Assume
that the world begins in period \((t-1)\) and ends immediately after period \((t+1)\). Suppose that we can measure the well-being of individual \(i\) in period \(t\) as \(\omega_{it}\), with \(\omega_{it} = 0\) if individual \(i\) is not alive in period \(t\). The well-being of the three individuals follows the trajectories that are summarized in Table 1.

\[
\begin{array}{c|ccc}
\text{ind. } i & t-1 & t & t+1 \\
\hline
\omega_{i(t-1)} & \omega_{it} & 0 & \\
\omega_{it} & 0 & \omega_{jt} & 0 \\
\omega_{k(t-1)} & \omega_{kt} & \omega_{k(t+1)} & \\
\end{array}
\]

Table 1: Three life courses

The overall lifetime social welfare can be written as a double aggregation, one across individuals and one across periods (the rows and columns of Table 1, respectively):

\[
SW = \mathcal{S}(\omega_{i(t-1)}, \omega_{it}, 0, 0, \omega_{jt}, 0, \omega_{k(t-1)}, \omega_{kt}, \omega_{k(t+1)}) .
\]  

(20)

The lifetime prioritarian approach takes into account the well-being of individuals over their whole lifetime and assumes separability between individuals. Hence, the social welfare function has the following separable structure\(^7\)

\[
SW^1 = \mathcal{S}(w_i(\omega_{i(t-1)}, \omega_{it}, 0), w_j(0, \omega_{jt}, 0), w_k(\omega_{k(t-1)}, \omega_{kt}, \omega_{k(t+1)})).
\]  

(21)

One first calculates the lifetime well-being for each individual, and then aggregates these lifetime well-being levels through a social welfare function of the prioritarian type. Applied to Eq. (1), this means that we would interpret \(w_{it}(\cdot)\) as the lifetime well-being of the individual \(i\) at time \(t\).

Yet this is not the common practice in empirical work, which is mostly restricted to cross-sectional data for a given period. This means that one changes the sequence of the aggregations, by first aggregating across individuals and then across periods. This period-by-period approach assumes another separable structure for \(\mathcal{S}(\cdot)\):

\[
SW^2 = \mathcal{S}(S(\omega_{i(t-1)}, 0, \omega_{k(t-1)}), S(\omega_{it}, \omega_{jt}, \omega_{kt}), S(0, 0, \omega_{k(t+1)})),
\]  

(22)

so that it becomes possible to focus on each period separately. The individual well-being levels in the analysis then become period-specific values, not the lifetime well-being levels. This has important consequences in a prioritarian setting. Indeed, it implies that the relative

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\(^7\)The subscripts attached to \(w(\cdot)\) indicate that individuals may have different preferences about the relative weight given to different periods of their life in the evaluation of overall lifetime well-being.
weights given to the well-being of the different individuals in period $t$ are independent of their well-being in the other periods. An individual who reaches a high well-being level in period $t$, but a very low one in the rest of his life, would get the same weight in $t$ as an individual who maintains the same high well-being level during his entire life. Differences in longevity are a striking example. In the simple example of Table 1, it is possible that individual $j$ has a very high well-being in period $t$, although his life is shorter than that of the other two individuals. This result may go against the fairness intuition of many prioritarians.\footnote{In fact, combining equations (21) and (22) implies that the function $\mathcal{S}(.)$ is additive in the temporal well-being levels, clearly illustrating that it becomes difficult to introduce lifetime considerations.}

In principle, we should therefore work with the aggregation sequence underlying Eq. (21): first aggregate across periods for each individual and then across individuals. Yet, this approach faces at least three difficulties in an empirical application. First, how to introduce differences in longevity in the analysis? In fact, this is a typical example of cumulative deprivation, as the economically weakest groups on average also have a shorter life. Second, there is a practical data problem. The available data rarely contain complete information on the past situation of the individuals. Third, there is a conceptual issue. When we calculate social welfare in period $t$, we often do not know the future life course of all individuals. Hence, our estimates of individual well-being are truncated. What is worse, the moment of truncation depends on the age of the individuals at the time of the analysis. Summarizing, we need an exceptionally rich dataset to implement Eq. (21).

How can we proceed in practice? One possibility is suggested by the so-called non-anonymous approach to measuring economic growth. In fact, the term “non-anonymous" is a misnomer, since the basic idea of this approach is to take the starting positions of the individuals into account when evaluating social progress. Loosely speaking, this can be seen as a correction on the weights that introduces information on the past life course of individuals and, hence, a lifetime perspective in the analysis. In this approach, the growth in social welfare $GR_t$ is defined as a weighted average of the growth rates in the well-being of the individuals ($gr_{it}$), with weights that are decreasing with their initial well-being level. A typical example is the measure proposed by Genicot and Ray (2013):

$$GR_t = \sum_{i=1}^{N} \left[ \frac{(w_{it})^{-\varepsilon}}{\sum_{j=1}^{N} (w_{jt})^{-\varepsilon}} \right] \times gr_{it}. \quad (23)$$

This measure gives a larger weight to the growth of well-being of those that start at a lower initial level of well-being. The weights become more sensitive to the initial distribution as the parameter $\varepsilon$ grows. When $\varepsilon = 0$, social progress reduces to the unweighted average of the individual growth rates and the past does not matter. Note that the weights in Eq. (23)
are similar to the weights in the Atkinson-approach to the relative growth incidence curve (see Eq. (13)). Of course, the growth rates to which these weights are applied are defined differently: Eq. (13) includes the growth rates at each percentile, in Eq. (23) we have the growth rates for each individual \(i\). By incorporating the past into the analysis, this non-anonymous approach gives us some idea about how a lifetime approach could be different from a temporal approach.

A second (pragmatic) approach is to focus on representative individuals within a cohort and introduce life expectancy in the measure of individual well-being. A simple approach to this issue has been proposed (among others) in the paper by Becker et al. (2005). Dropping the subscript \(i\) for notational convenience, they write the indirect utility function \(V(Y, \sigma)\) of an individual with lifetime income \(Y\) and survival function \(\sigma\) as:

\[
V(Y, \sigma) = \max \int_0^\infty \exp(-\rho t)\sigma_t u(c_t) dt
\]

subject to

\[
Y = \int_0^\infty \exp(-rt)\sigma_t y_t dt = \int_0^\infty \exp(-rt)\sigma_t c_t dt,
\]

where \(c_t\) is consumption in period \(t\), \(u(\cdot)\) a utility function and \(r = \rho\) the interest rate and time preference. This indirect utility function can be seen as a simple example of the lifetime well-being function that was introduced in Eq. (21), in which only two attributes are considered: consumption and life expectancy.

Let us now consider the special case in which the individual receives the same income \(y\) in all years of his life. With the budget constraint (25), this implies that consumption also remains constant, i.e., that \(c_t = c = y\). The indirect utility function (24) can then be written as

\[
V(y, \sigma) = u(y)L(\sigma),
\]

with \(L(\sigma) = \int_0^\infty \exp(-rt)\sigma_t dt\). Eq. (26) nicely separates instantaneous utility \(u(y)\) and the utility related to longevity \(L(\sigma)\). Of course, this simple decomposition no longer holds if we consider the allocation of more attributes over the whole life-cycle or specify a more realistic income-generating process. Moreover, life expectancy is an \textit{ex ante} concept and the actual length of life may be very different for different individuals within a group with the same life expectancy.
2.4.2 Variable population

In the previous sections we have assumed that the population size, $N$, remains constant. In most applications, however, $N$ may vary. When comparing the social condition of different countries, we have to take differences in their population size into account. When looking at changes over time of the social condition of a given country, again the size of the population of that country may change. Moreover, in both comparisons the people making up the society are not the same. This is obvious for international comparisons. In comparisons over time, some individuals in the initial situation will have died, and the final situation will also include newborns. This “non identity” problem has been discussed at great length in the philosophical literature in the wake of Parfit (1984) but we leave it aside in this chapter. Instead, we focus on the issue of how to handle different populations sizes in the measurement of social welfare (this issue is also discussed in chapter 2, this volume).

Implicitly, we have already addressed the issue by taking in Eq. (1) the average and not the sum of the (transformed) well-being levels. This approach could be called “average prioritarianism”. If we had taken the sum (what could be called “total prioritarianism”), it would be easier for larger countries to be better off than smaller countries. In fact, total prioritarianism (like total utilitarianism, which has been the main focus of the literature) implies what Parfit (1984) has called the “repugnant conclusion”: for any society in which individuals are at a high level of well-being, there is a second society that is ranked higher in terms of social welfare and in which everyone is at a very low level of well-being. Average prioritarianism avoids that implication, but it has other debatable consequences. Adding a person to a rich society (with a high average individual well-being) decreases social welfare if the well-being of that individual (weighted by the function $g(.)$) is smaller than the overall average.\(^9\) As a specific example, low-skilled immigration in a rich country is evaluated as negative for the social condition of that country, even if these migrants reach a decent level of well-being, as long as that level of well-being is below the average in the receiving country. On the other hand, in a very poor country, with an average below the subsistence level, adding another person below that subsistence level (but above the average), could be welfare improving.\(^10\) Neither of these two implications is very attractive from a welfare point of view. The latter problem is avoided by an alternative approach, which could be called “critical-level prioritarianism” (Blackorby et al. 2005) in which Eq. (1) is replaced by

$$
S_{CRIT}(a_t) = \sum_{i=1}^{N} (g(w_i(a_t)) - g(w_{CRIT})).
$$

\(^9\)This is a variant of what is called in the literature for a utilitarian framework the “mere addition principle”.

\(^10\)This has been called in the literature the “negative expansion principle”.

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Critical-level prioritarianism also avoids the repugnant conclusion. However, it still implies that adding a person with a well-being level below $w^{\text{CRIT}}$ would lead to a decrease in social welfare, which is not very attractive if $w^{\text{CRIT}}$ is large.

The literature on population ethics is rapidly growing and at this stage it has not yet reached a consensus. The issue is largely neglected in the applied work, however, where “per capita” (average) approaches, such as Eq. (1) take almost a monopoly position.

3 Applications: social welfare with different measures of individual well-being

The prioritarian approach to social progress, as described in the previous section, can be implemented with different concepts of well-being. The use of different concepts may lead to different results. Inevitably, hard normative and modelling choices are needed to do applied work. In this section, we give a brief overview of the three main approaches to the measurement of well-being: objective, subjective and preferentialist. For further details we refer to chapter 3 in this book by Adler and Decancq. The focus of this book and of the empirical illustration in Section 4 is on the preferentialist approach, but we also discuss the other two approaches, since they are prominently present in the applied literature. For each of the three approaches, we discuss some illustrative applications. While the applied literature is large, only very few studies explicitly implement a prioritarian social welfare function. The aim of this section is not to be complete, but to illustrate some normative and modelling choices with examples. While we refer sporadically to inter-country comparisons, our main focus is on the evaluation of changes of social conditions over time, i.e., on measuring social progress.

We only discuss approaches that implement an explicit and inclusive measure of individual well-being. Other methods are found in the applied literature. One is the calculation of multidimensional inequality measures (see Weymark (2006) for a survey). Combining a multidimensional transfer principle with other axioms (e.g. scale invariance) yields inequality measures that implicitly suggest a functional specification for $S(a_t)$ and $w(a_{it})$, with the latter assumed to be the same for all individuals. Unfortunately this justification is not rooted in an explicit theory of individual well-being. Another method is to introduce richer information in the social evaluation by constructing a “dashboard” that collects information on various one-dimensional outcomes and their evolution over time (see Atkinson et al. (2002) for a dashboard of social indicators to monitor the performance of the EU with respect to social inclusion, for instance). Although useful to monitor dimension-specific policy outcomes,
we consider the dashboard approach a non-starter for the measurement of social condition and social progress. The reason is that it remains blind to the phenomenon of “cumulative deprivation” or, more generally, the correlation between the different dimensions of well-being (see Decancq and Schokkaert (2016) for a more extensive argument and Ravallion (2011) for a critique of this position).

3.1 Objective measures of well-being

3.1.1 Income

The most popular approach in applied work remains to look at income $y_{it}$ as a measure of well-being.\footnote{Prioritarian social welfare functions are defined over \textit{individual} well-being levels. Much of the empirical work, however, relies on \textit{household-level} data (e.g. household income, household consumption), mainly because of data limitations. Traditionally, equivalence scales are used to “equivalize” household income (or consumption) into individual income (or consumption). Underlying the use of equivalence scales is the assumption that household income (and hence, in this narrow framework, well-being) is distributed equally within the household. This is definitely not a realistic assumption. Recently, the theory of household behaviour has moved from the unitary approach (the household as one unit) to the collective approach (the household as a collective of individuals), allowing for the separate evaluation of the well-being of the different household members (Chiappori and Meghir, 2015). To the best of our knowledge, this approach has not yet led to applications from a prioritarian perspective.} In the utilitarian interpretation of Eq. (3), the social welfare function then becomes (some variant of) per capita GDP and social progress becomes GDP-growth. In a prioritarian interpretation, inequality can be introduced as in equations (9) or (10). Interestingly, Atkinson himself presented an empirical comparison of 12 countries in his seminal paper (Atkinson (1970), pp. 258-261). He compared seven advanced and five developing countries based on the equally-distributed equivalent of their income distribution for several values of the inequality aversion parameter. Countries with a high average income perform better when the inequality aversion parameter is low and countries with a more equal income distribution perform better when the parameter is high. The application illustrates that the equally-distributed equivalent is a flexible tool to make international comparisons. In its wake, many studies have implemented the simple decomposition (9), but, to the best of our knowledge, only few of them have interpreted their results explicitly within a prioritarian framework. Empirical work using Kolm’s prioritarian social welfare function is even more scarce.

The (relative) growth incidence curve (see Section 2.2.2) has also been mainly applied in an income setting, for which it was introduced from the very start. A recent famous example of its use is the “Elephant curve” proposed by Lakner and Milanovic (2016), showing the income growth at different percentiles of the global income distribution (see Figure ??). The curve illustrates the relative decline in the income of the middle class in the richer countries.
Income is relatively easy to measure. Moreover, focusing only on income means that one does not have to tackle the thorny issue of how to aggregate the different attributes in one measure of individual well-being. Yet, this is precisely its main weakness. Most people would agree that income is not the only relevant attribute of individual well-being and that non-monetary attributes (such as health, job characteristics and social relations) are essential components of a good life too.

3.1.2 Multidimensional composite indices of well-being

Some scholars have claimed that individual well-being or individual flourishing is an objective notion that does not depend on individual preferences (see Chapter 3, this volume for an overview and discussion). They argue that preferences can be misguided, biased, or even ill-defined. In an objective approach, the weighting of the different attributes is decided by the analyst, not by the individuals themselves - and is usually assumed to be uniform for all individuals. Without necessarily referring to a sophisticated theory, much of the applied work has - for practical reasons - also resorted to an objectively given weighting scheme (see Decancq and Lugo (2013) for an overview of different methods to select a weighting scheme).\textsuperscript{12}

Two international institutions are particularly active in the production of such objective indices of well-being: the UNDP and the OECD. The UNDP computes the “Human Development Index” (HDI) annually since the early 1990s and, more recently, the OECD has

\textsuperscript{12}Such an objective, “perfectionist” approach is often followed in the literature on educational policies - see chapter 9, this volume.
launched its “Better Life Index” (BLI).\footnote{For the Better Life Index, see http://www.oecdbetterlifeindex.org.} Whereas the HDI covers all countries of the world for a limited number of dimensions, the BLI covers only the OECD member countries for a broader set of dimensions of well-being. To be more precise, the HDI consists of three dimensions: standard of living, education and life expectancy. The BLI consists of 11 dimensions grouped in two categories: quality of life and material conditions. Quality of life is measured by eight dimensions: health status, work life balance, education and skills, social connections, civic engagement and governance, environmental quality, personal security and subjective well-being. Material conditions are measured by three dimensions: income and wealth, jobs and earnings and housing.

Neither the HDI nor the BLI uses data at the individual level, and they do not include any distributional concerns. Hence, it may be best to interpret them as (objective) specifications of the well-being function of a representative individual \( i \) in the various countries:

\[
w_{it}^{\text{obj}} = w(a_{it}) = \left( \sum_{j=1}^{m} \omega_j \times a_{ijt}^{1-\mu} \right)^{1/(1-\mu)}.
\]  

(28)

In this mathematical structure two sets of parameters need to be selected; a weight \( \omega_j \) for each dimension \( j \) and a parameter \( \mu \) that captures the degree of complementarity between the dimensions of well-being. For the HDI all weights are fixed at 1/3, and after its revision in 2010 the complementarity parameter \( \mu \) equals 1 (in which case Eq. (28) becomes multiplicative). For the BLI, the weights can be selected by the user in an online web application and the parameter \( \mu = 0 \).

One can of course also see Eq. (28) as the specification of well-being for each individual separately, and then aggregate all individual well-being levels through a prioritarian social welfare function. As an example, Bosmans et al. (2015) present the following specification, which introduces Eq. (28) in (the equally-distributed equivalent of) an Atkinson social welfare function:

\[
\tilde{S}(a_t) = \left( \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{j=1}^{m} \omega_j \times a_{ijt}^{1-\mu} \right)^{1-\gamma/(1-\mu)} \right)^{1/(1-\gamma)}.
\]  

(29)

This specification contains an additional parameter \( \gamma \) to capture the degree of inequality aversion. The special case in which \( \gamma = \mu \) has been proposed by Foster et al. (2005) and Alkire and Foster (2010) to derive the inequality-adjusted Human Development Index, which is published every year in the Human Development Report since 2010. Decancq (2017) argues in favour of keeping both parameters to have the flexibility to capture the degree of complementarity (\( \mu \)) and the aversion to inequality (\( \gamma \)) independently. He proposed a
distribution-sensitive Better Life Index, based on an Atkinson social welfare function, and applies it to a synthetic micro data set for all OECD member countries. His work illustrates how the welfare ranking of countries crucially depends on the inequality aversion parameter: as an example, while the United States obtains a higher social welfare for low values of the parameter $\gamma$, Austria scores better for more inequality averse social welfare function.

Decancq and Ooghe (2010) use the framework of Eq. (29) to study the question whether the world moved forward. Since their analysis is at the country level, they do not take up within-country inequality, but they do integrate in their analysis between-country inequality in well-being. Analogously to what we have discussed in Section 2.2.1, they decompose Eq. (29) to obtain a measure of multidimensional inequality $I(a_t)$

$$\tilde{S}(a_t) = \tilde{S}(\bar{a}_t) \times \left[ 1 - \left( 1 - \frac{\tilde{S}(a_t)}{\tilde{S}(\bar{a}_t)} \right) \right]$$

(30)

where $\bar{a}_t$ is the matrix that contains the average outcomes in each dimension. They consider the tree dimensions of the HDI and fix the parameter $\mu = 1$.\textsuperscript{14} They perform a sensitivity analysis for the role of the aversion to inequality ($\gamma$) and the different possible weighting schemes. They find that between-country social welfare has increased between 1980 and 2008. When focusing on between-country multidimensional inequality, they find that if life expectancy or standard of living get a large weight, the inequality trend depends on the level of inequality aversion.

### 3.2 Happiness and life satisfaction

Completely neglecting the differences in the relative importance that people attach to the various life dimensions goes against the economic tradition of “consumer sovereignty”. In recent years, this criticism on the objective approach has been vigorously restated by researchers active in the field of measuring happiness. Layard (2005, p. 121) puts it bluntly: “Ethical theory should surely focus on what people feel, rather than what people think is good for them. If we accept the Marxist idea of false consciousness’, we play God and decide what is good for others, even if they will never feel it to be so.”. Layard himself is advocating a subjective well-being approach, in which well-being is reduced to a mental state such as happiness. This emphasis on feelings brings us back to a version of Benthamite utilitarianism. With one important difference, though: “individual happiness” can be measured in

\textsuperscript{14} This restriction follows from imposing a strong invariance principle, see also Tsui (1995).
large-scale surveys and is therefore readily available as a measure of individual well-being. These large-scale surveys typically also inquire how satisfied people are with their lives, all things considered. Although the results on this question are highly correlated with reported feelings of happiness, conceptually they are different since life satisfaction refers to a more cognitive (albeit still subjective) evaluation of life. It seems natural to interpret “happiness feelings” as one possible dimension of life (one element in the vector $a_{it}$) and “life satisfaction” as an overall evaluation of the attributes contained in $a_{it}$. This interpretation brings us close to the preferentialist approach.

Recently, the literature on happiness and life satisfaction is booming. The bulk of the applied work looks at average happiness or life satisfaction, and can therefore be interpreted as an application of average utilitarianism. A calculation of averages is only meaningful if one assumes that happiness or life satisfaction can be measured at a cardinal level and that the response scale is used in the same way by all respondents, so that the resulting values are interpersonally comparable. Yet, the data on happiness or life satisfaction are typically measured on an ordinal scale (see Ferrer-i-Carbonell and Frijters (2004) for a discussion). Moreover, there is evidence of heterogeneous use of the response scale by different respondents (see Beegle et al. (2012), Angelini et al. (2014), Ravallion et al. (2016) and Fleurbaey and Blanchet (2013) for a more general theoretical analysis). If these interpersonal differences reflect differences in expectations and aspirations, it is debatable whether they should be respected in interpersonal well-being comparisons: if two individuals are in the same objective situation and have identical preferences, but one is less happy (or less satisfied) because she has more demanding aspirations, should she get a larger weight in a prioritarian evaluation of social states? One could perhaps argue that differences in scaling reflect individual idiosyncrasies that are likely to be wiped out in the averaging process. This does not solve the problem that the cardinalization is an arbitrary one, however. Moreover, even if we can see the response scale differences as noise, they cannot be neglected for interpersonal comparisons at the individual level. As an example, it has been shown that they have a strong influence on the identification of who are the poor in a given society (see, e.g., Decancq et al. (2015b)).

Most of the applied empirical literature neglects these measurement issues and analyses the happiness and life satisfaction data as if they were cardinally measurable and interpersonally comparable. An example is offered by the yearly published “World Happiness Reports” (Helliwel et al., 2019). In the period 2016-2018, average happiness was the highest in Fin-

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15 King et al. (2004) have proposed the use of vignettes to model and correct for differential use of the response scale, see also Angelini et al. (2014).

16 We focus here only on the measurement issues. Even if the measurement were perfect, the use of happiness or life satisfaction as measures of individual well-being would still raise serious ethical issues. See the seminal analysis in Sen (1985) and Dworkin (1981) and chapter 3 in this volume.
land, Denmark and Norway and the lowest in South Sudan, the Central African Republic and Afghanistan. In the period from 2005-2008 to 2016-2018, the largest increase in average happiness has been realized in Benin, Nicaragua, Bulgaria and Latvia, the largest decrease in average happiness is found in (not surprisingly) Syria and Venezuela - both with a large decrease of almost two points on a 10 point-scale. The authors also report results on the inequality of happiness, measured by the standard deviation of the distribution of the individual evaluations of life satisfaction (also on a 10 point-scale). Comparing the results for different regions in the world yields interesting insights. As an example, they find that in the period 2006-2018 average life satisfaction has remained almost constant in Sub-Saharan Africa, but has declined considerably in the Middle East. In the same period inequality in life satisfaction has increased in the two regions and very strongly so in Sub-Saharan Africa. Some authors have calculated the inequality in life satisfaction while taking due account of the ordinal nature of the data (see, e.g., Dutta and Foster (2013)), but to the best of our knowledge, there are no applications in which hedonic measures have been used in a prioritarian social welfare function as given by Eq. (1).

3.3 Preferentialist approaches

In a preference-based approach, the well-being measure is a representation of the individual preferences. If we denote the preference relation of individual $i$ by $R_i$, preferentialist measures of well-being satisfy the condition\(^{17}\)

$$w_i(a_{it}) \geq w_i(a'_{it}) \iff a_{it} R_i a'_{it}. \quad (31)$$

All functions $w(\cdot)$ that satisfy Eq. (31) are representations of a preference relation and, hence, utility functions. The ethical issue is now to choose between different possible cardinalizations. In fact, if one accepts that the answers to a life satisfaction question reflect individual preferences, life satisfaction is one of these possible cardinalizations. As is discussed in Section 3.2, it raises some ethical questions, however.

The literature has suggested many other possibilities with a specific ethical rationale for the chosen cardinalization (see chapter 3, this volume, for a more extensive discussion).\(^{18}\) In Section 3.3.1, we discuss money metric utilities, which have been particularly influential in

\(^{17}\)This condition is called the Deference Principle in chapter 3 of this volume.

\(^{18}\)The equivalent income is one possible choice within the “equivalence approach” (Fleurbaey and Maniquet (2011). Another popular family of well-being measures within the same approach is that of ray utilities (Deaton, 1979; Fleurbaey and Tadenuma, 2014). Note that Eq. (32) implicitly defines an “instantaneous” equivalent income for period $t$. The extension to a life-cycle equivalent income is straightforward.
consumption theory. Going beyond market outcomes, we discuss in Section 3.3.2 the so-called equivalent income.

The equivalent income $y^*_it$ is defined implicitly by the Eq.:

$$ (y^*_it, \tilde{x}_{it}) I_i (y_{it}, x_{it}), $$(32)

where $\tilde{x}_{it}$ is a vector with reference values for the non-income attributes. We formulate these reference values here in a flexible way as dependent both on the individual and on the time period concerned. In empirical applications it is common to assume that these reference values are constant across individuals and time, so that $\tilde{x}_{it} = \tilde{x}$. It is easily seen that

$$ y^*_it = y_{it} - WTP_i(x_{it} \rightarrow \tilde{x}_{it}), $$

where $WTP_i(x_{it} \rightarrow \tilde{x}_{it})$ denotes the willingness-to-pay of individual $i$ for a move from $x_{it}$ to $\tilde{x}_{it}$.

Equivalent incomes respect individual preferences in a risk-free setting, but they do not take up individual differences in risk aversion. An alternative cardinalization, going back to the seminal work of Harsanyi (1977) and recently advocated by Adler (2016), makes use of von Neumann-Morgenstern (vNM) utility functions (see chapter 3, this volume).

Preference-based approaches raise a challenging implementation issue: how to estimate preferences over attributes? If we only consider market commodities (or, more precisely, attributes that can be chosen by the individuals), revealed preference techniques can be used. As soon as we introduce non-market commodities (such as health) where freedom of choice is limited, one has to resort to stated preferences (see, e.g., Fleurbaey et al. (2013)) or to deriving ordinal information from life satisfaction data (see, e.g. Decancq et al. (2015b)). The latter method is used in the empirical illustration in Section 4. It retrieves (ordinal) information about the marginal rates of substitution from an estimated life satisfaction equation. This ordinal information allows to estimate equivalent incomes. Indeed, if we write the life satisfaction equation as $SWB_{it} = f_i(y_{it}, x_{it}, u_{it})$ with $x_{it}$ capturing the non-income attributes of well-being and $u_{it}$ an error term with zero mean, the equivalent income can be derived from solving the following implicit equation:

$$ f_i(y_{it}, x_{it}, u_{it}) = f_i(y^*_it, \tilde{x}_{it}, 0), $$

with $\tilde{x}_{it}$ the reference values for the non-income attributes.$^{19}$ In the case of vNM utilities,$^{19}$ Eq. (34) makes the life satisfaction equation individual-specific. Of course, in empirical work with the available panel data, the degree of inter-individual variation has to be restricted.
one needs additional information about risk aversion. We return to the implementation of the vNM approach in our empirical application of Section 4.

Many of the cross-sectional studies attempt to take into account preference heterogeneity. However, when analysing changes over time, it is usually assumed that preferences do not change. This is obviously not realistic, but introducing changing preferences into the analysis raises challenging normative and empirical questions (see Fleurbaey and Tadenuma (2014) for a discussion).

### 3.3.1 Market commodities

The preference-based approach has taken a prominent place in theories of social welfare based on consumption theory (see, e.g., Deaton and Muellbauer (1980)). A typical example is the work of Jorgenson and co-authors. Denote by $q_{it}$ the quantities of market commodities consumed by household $i$ in period $t$, and write $U_i(q_{it}, A_i)$ as the utility representation of the preferences of household $i$ with characteristics $A_i$ over market commodities. To estimate the preference parameters, Jorgenson (1990) assumes that household preferences can be represented by a translog indirect utility function. Since he focuses on market commodities only, all the parameters in that function can be estimated from household consumption data. To move to social welfare, Jorgenson and Slesnick (2014) propose the following social welfare function:

$$SW(V_1, \ldots, V_N) = \ln V - \gamma(q) \left[ \frac{\sum_i m_0(p, A_i) |\ln V_i - \ln V|^{-\rho}}{\sum_k m_0(p, A_k)} \right]^{-1/\rho},$$

(35)

with $V_i$ the translog indirect utility function and $m_0(p, A_i)$ the general household equivalence scale. Eq. (35) can be interpreted as follows. The first term $\ln V_i = \sum_i m_0(p, A_i) \ln V_i$ captures average social welfare, while the second term is a measure of inequality. It is a homogeneous function of the deviations of the individual welfare levels from the average. The parameter $\rho$ measures the degree of inequality aversion: if $\rho = -\infty$, the dispersion term vanishes and social welfare reduces to the simple average of the logarithm of individual well-being. Jorgenson and Slesnick call this the “utilitarian” case, while they denote the case of $\rho = -1$ as “egalitarian”.

From the perspective of the rest of the literature, the social welfare specification (35) is rather idiosyncratic.\footnote{For Eq. (35) to satisfy the Pareto-principle, the function $\gamma(q)$ has to be restricted. Jorgenson and Slesnick (2014) assume that it takes the maximum value consistent with the Pareto principle.} However, it is not difficult to use the individual welfare levels, as estimated from consumption behaviour, in another social welfare function. One possibility, mentioned
in Jorgenson and Schreyer (2017), is the Atkinson-specification (as in Eq. (9)).

The strengths and limitations of this approach are both linked to the fact that the authors only consider market commodities. This makes it possible to estimate and implement the revealed preference approach. When working with individual household data, it becomes possible to introduce (and estimate) a considerable amount of preference variation: in addition to household size, the vector $A$ includes age and gender of the household head, and urban versus rural residence. Moreover, the focus on consumption allows for the integration of equity considerations in the regular National Accounts framework. This is one of the main aims of the recent work by Jorgenson and Schreyer (2017). The application of their methodology to the US data for the period 1948-2010 shows a positive contribution of the change in equity for the period 1948-1973. However, since the mid-seventies the equity component has turned negative and the growing inequity has led to a decrease in the growth rate of social welfare.

Despite these practical advantages, it remains difficult to accept that individual well-being and social progress would only depend on the consumption of market commodities. We now turn to approaches that have tried to incorporate other, non-material, considerations.

### 3.3.2 Including non-market commodities

As soon as non-market commodities are included in the vector $a_{it}$, the estimation of preferences becomes a real challenge. We first present some studies that have tried to circumvent the estimation of the preferences by a calibrating them. We then discuss an application in which preferences are estimated on the basis of a life satisfaction equation. This approach is also used in the empirical illustration in Section 4.

**Calibrated preferences** A central non-market commodity is health. As described in Section 2.4.1, Becker et al. (2005) have proposed to introduce longevity into the analysis by specifying a country level indirect utility function $V(y_t, \sigma_t) = u(y_t)L(\sigma_t)$, where $y_t$ is GDP per capita, $\sigma_t$ the survival function of the country in period $t$ and $L(\sigma_t) = \int_0^\infty \exp(-rt)\sigma_t dt$ (see Eq. (26)). In their empirical analysis, the authors assume a constant relative risk aversion specification for $u(\cdot)$ to arrive at:\textsuperscript{21}

$$V(y_t, \sigma_t) = \left( \frac{y^{1-1/\gamma}}{1 - 1/\gamma} + \alpha \right) \times L(\sigma_t).$$

\textsuperscript{21}The constant $\alpha$ is used to obtain a realistic calibration of the value of longevity.
The authors calibrate the parameter values of this Eq. based on findings from the literature about relative risk aversion and the value of life (they set $\alpha = -16.2$, $\gamma = 1.250$ and $r = 0.03$). Based on Eq. 36, they compute for each country a full income $y^*$ so that:

$$V(y^*, \sigma_t) = V(y_{t+1}, \sigma_{t+1}).$$ (37)

The growth rate rate $(y^* - y_t) / y_t$ captures both the increase of GDP and the improvement in longevity and is their measure of social progress between the periods $t$ and $t+1$. Becker et al. (2005) implement this approach at the country level, so that their results are insensitive to the within-country distribution of full income. They focus on the inequality in social progress between countries. Their results show that improvements in longevity have contributed considerably to growth in full income, especially for the poorest countries.

Jones and Klenow (2016) extend the framework of Becker et al. (2005) substantially. They include the disutility of labour and use consumption rather than income. Their specification of flow utility is

$$u(C, l) = u + \log(C) - \frac{\theta \epsilon}{1 + \epsilon}(1 - l)^{1+\epsilon},$$ (38)

where $C$ is consumption, $l$ is leisure, $\theta$ is the weight given to leisure and $\epsilon$ is the Frisch-elasticity of labour supply. Longevity is entered as in Eq. (26). Importantly, in their empirical work they make use of individual survey data, so that inequality can also be taken into account. Like Becker et al. (2005), they implement a consumption equivalent measure to answer questions as: what proportion of consumption in the United States, given the US values of leisure, mortality and inequality, would deliver the same utility level as the values in France? Since they calculate utility for each country, their social welfare function is utilitarian.

After calibrating all the parameter values in the model, Jones and Klenow (2016) find that the level of social welfare in Western Europe is much closer to that in the United States than it would appear from looking at GDP per capita alone. Longer lives with more leisure time and more equal consumption in Western Europe largely offset their lower average consumption. In addition to these country level comparisons, they also calculate social progress as the growth rates of the consumption equivalents over time. Some detailed results are shown in Table 2. Life expectancy is again the main driver of the differences between welfare growth and income growth.

Both Becker et al. (2005) and Jones and Klenow (2016) have assumed preference homogeneity and calibrated a single indirect utility function for a representative agent for each country. Fleurbaey and Gaulier (2009) calculate equivalent incomes with reference values as they are defined in Eq. (32). They extend the parametric specification given in Eq. (36) by including
Welfare
Income
Diff.
Decomposition of difference

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<td>1.04</td>
<td>0.10</td>
<td>-0.05</td>
<td>-0.16</td>
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<tr>
<td>(1984-2005)</td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>US</td>
<td>3.09</td>
<td>2.11</td>
<td>0.98</td>
<td>0.89</td>
<td>0.51</td>
<td>-0.10</td>
<td>-0.24</td>
<td>-0.08</td>
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<tr>
<td>(1984-2006)</td>
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</tr>
<tr>
<td>Italy</td>
<td>2.73</td>
<td>2.02</td>
<td>0.72</td>
<td>1.33</td>
<td>0.03</td>
<td>-0.17</td>
<td>-0.24</td>
<td>-0.22</td>
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<tr>
<td>(1987-2006)</td>
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<td>0.39</td>
<td>2.25</td>
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<td>0.81</td>
<td>0.18</td>
<td>-0.16</td>
<td>-0.00</td>
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<tr>
<td>(1993-2006)</td>
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<td></td>
<td></td>
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<tr>
<td>Mexico</td>
<td>1.87</td>
<td>1.05</td>
<td>0.82</td>
<td>1.09</td>
<td>0.26</td>
<td>-0.23</td>
<td>-0.16</td>
<td>-0.14</td>
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</tr>
<tr>
<td>(1984-2006)</td>
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<td></td>
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<tr>
<td>Average</td>
<td>3.14</td>
<td>2.13</td>
<td>1.02</td>
<td>1.15</td>
<td>0.11</td>
<td>-0.05</td>
<td>-0.16</td>
<td>-0.04</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Income and welfare growth (Source: Jones and Klenow (2016))

the disutility of labour, the risk of unemployment and household composition. They allow for inter-country differences in preferences for leisure by taking the observed net wage rate as an estimate of the marginal rate of substitution between consumption and leisure. The resulting equivalent incomes are then introduced in an Atkinson prioritarian social welfare function which penalizes average full income for the inequality in its distribution. The penalty for inequality in the distribution of full incomes is approximated by the inequality in the distribution of ordinary incomes. Their empirical work focuses on instantaneous comparisons of different countries, rather than the measurement of social progress. The authors implement their approach for 24 OECD member countries and find that the obtained ranking of countries differs when the social welfare function becomes inequality averse ($\gamma = 1, 5$) compared to the utilitarian (inequality-neutral) case. Especially Japan and the northern European countries improve their position, while Ireland, the UK, the US, Australia, Portugal and Italy worsen their position.

**Estimated preferences**  As mentioned before, preferences for non-market commodities can also be estimated on the basis of a life satisfaction equation, rather than calibrated. This approach is illustrated by Decancq and Schokkaert (2016), who have estimated a life satisfaction equation with pooled information from the European Social Survey for 2008 and 2010 and for 15 EU-members, Switzerland, Norway and the Russian Federation. The
attributes included in the analysis are income, health, employment, social relations and safety. Interaction effects between demographic variables and attributes are introduced to incorporate intergroup preference heterogeneity. Decancq and Schokkaert find, e.g., that higher educated respondents give a higher weight to income and a lower weight to health. The estimated parameters of the life satisfaction equation are then used to estimate equivalent incomes for all individuals in the sample on the basis of Eq. (34).

Decancq and Schokkaert (2016) use a rank-dependent generalized Gini social welfare function and decompose its equally-distributed equivalent for a country in period $t$ as follows:

$$\tilde{S}_{\rho}^{DS}(a_t) = \mu(a_t)[1 - GI_{\rho}(a_t)], \quad (39)$$

where $\mu(a_t) = \sum_i (w_i(a_{it})) / N$ is the average equivalent income in period $t$ and $GI_{\rho}$ is a generalized Gini inequality measure with $\rho$ a parameter of inequality aversion (with $\rho = 1$, $GI_1 = 0$ and social welfare reduces to mean individual well-being, with $\rho = 2$, $GI_2$ becomes the well-known Gini coefficient). While the decomposition of Eq. (39) resembles (9), the social welfare function does not have the separable structure of Eq. (1) and, hence, their approach is not generalized utilitarian (see chapter 2, this volume, for a discussion).

They find that the introduction of health as a dimension of well-being has a strong effect on country rankings based on social welfare. The growth rate of social welfare is summarized in Table 3. It shows that the inter-country ranking of the growth rates of social welfare (“social progress”) is crucially affected both by the move from income to equivalent income and by the introduction of a sensitivity to the distribution of well-being, i.e., by selecting the parameter of inequality aversion at a value larger than 1.

### 4 An empirical illustration: social progress in Russia

Between 1995 and 2005, the Russian economy underwent sharp changes, including a deep financial crisis in August 1998. One may wonder how much social progress there was over the considered decade, and for whom? We use three waves of the Russia Longitudinal Monitoring Survey (RLMS-HSE) in 1995, 2000 and 2005 to investigate this question and illustrate some issues that were discussed in this chapter. We measure social progress using the Atkinson and Kolm-Pollak social welfare function for several values of the inequality aversion parameters, relying on three different well-being measures: expenditures, equivalent incomes and vNM-utilities.

Because the shape of the distribution of these well-being measures differs considerably, it is a challenge to choose the inequality aversion parameters $\gamma$ and $\beta$ in a “comparable” way.
<table>
<thead>
<tr>
<th>Country</th>
<th>Income (%)</th>
<th>Equivalent income (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho = 1$</td>
<td>$\rho = 2$</td>
</tr>
<tr>
<td>BE</td>
<td>-0.55</td>
<td>-0.55</td>
</tr>
<tr>
<td>CH</td>
<td>7.35</td>
<td>10.18</td>
</tr>
<tr>
<td>CZ</td>
<td>-4.26</td>
<td>-5.42</td>
</tr>
<tr>
<td>DE</td>
<td>0.09</td>
<td>0.85</td>
</tr>
<tr>
<td>DK</td>
<td>-1.73</td>
<td>-2.53</td>
</tr>
<tr>
<td>ES</td>
<td>-2.24</td>
<td>-3.87</td>
</tr>
<tr>
<td>FR</td>
<td>-1.53</td>
<td>-1.02</td>
</tr>
<tr>
<td>GB</td>
<td>-2.16</td>
<td>0.36</td>
</tr>
<tr>
<td>GR</td>
<td>-5.81</td>
<td>-6.96</td>
</tr>
<tr>
<td>HU</td>
<td>-2.32</td>
<td>-2.65</td>
</tr>
<tr>
<td>NL</td>
<td>-2.03</td>
<td>-0.91</td>
</tr>
<tr>
<td>NO</td>
<td>-1.54</td>
<td>-2.92</td>
</tr>
<tr>
<td>PL</td>
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<td>2.72</td>
</tr>
<tr>
<td>RU</td>
<td>-4.69</td>
<td>-4.35</td>
</tr>
<tr>
<td>SE</td>
<td>-1.25</td>
<td>-1.46</td>
</tr>
<tr>
<td>SI</td>
<td>-4.5</td>
<td>-4.3</td>
</tr>
</tbody>
</table>

Table 3: Growth rates of income and equivalent income in period 2008-2010 (Source: Decancq and Schokkaert (2016))
We therefore consider a thought experiment with a leaky bucket transfer between a person at the 90th percentile of the well-being distribution and a person at the 10th percentile. We consider four cases in which \( \kappa \), the leak of the leaky bucket transfer, amounts to 0%, 50%, 90% and 99%, respectively. If the leak \( \kappa \) equals 99%, for instance, and the person at the 90th percentile sends a transfer of 100 ruble, only 1 ruble is received by the person at the 10th percentile. The remaining 99 ruble are lost. The leaky bucket transfer generates two opposing forces on social welfare: the leak reduces total well-being, but the transfer also makes the well-being distribution more equal. We select the inequality aversion parameters of the social welfare function so that both forces precisely compensate each other and social welfare remains the same before and after the leaky bucket transfer. When the social welfare-neutral leak is 0%, the social planner is not willing to accept any leak in compensation for the reduction in well-being inequality. Hence, the social welfare function that the planner is using is inequality-neutral. The larger the welfare-neutral leak \( \kappa \) is, the more weight the social planner gives to the reduction in well-being inequality, and, hence, the more inequality averse the social welfare function becomes. The implied inequality aversion parameters \( \gamma \) and \( \beta \) for the distribution of the considered well-being measures in 1995 are given in Table 4. The implied parameter values are quite different across the different well-being measures. These differences reflect the differences in the shape of the well-being distribution in 1995. The absolute difference between the 10th and 90th percentile of the distribution of expenditures is smaller than for equivalent incomes, for instance, whereas the ratio is larger (leading to larger \( \gamma \) values and smaller \( \beta \) values for the same \( \kappa \) value, respectively).

Table 5 and 6 present the equally-distributed equivalent of social welfare for the three considered well-being measures using the Atkinson social welfare function (Table 5) and the Kolm-Pollak social welfare function (Table 6) in 1995, 2000 and 2005. In the following sub-

---

**Table 4: Inequality aversion parameters \( \gamma \) and \( \beta \) for different values of the social welfare-neutral leak \( \kappa \)**

<table>
<thead>
<tr>
<th>( \kappa )</th>
<th>( \gamma )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50%</td>
<td>0.36</td>
<td>0.00008</td>
<td>0.15</td>
<td>0.00022</td>
<td>1.25</td>
<td>1.892</td>
</tr>
<tr>
<td>90%</td>
<td>1.21</td>
<td>0.00028</td>
<td>0.50</td>
<td>0.00074</td>
<td>4.15</td>
<td>6.287</td>
</tr>
<tr>
<td>99%</td>
<td>2.41</td>
<td>0.00056</td>
<td>0.99</td>
<td>0.00147</td>
<td>8.31</td>
<td>12.57</td>
</tr>
</tbody>
</table>

---

22 See also Chapter 2 of this book for a similar though experiment with a leaky bucket transfer.
23 The \( \gamma \) and \( \beta \) parameter values for the considered well-being measures are obtained by using Eq. (11) and equalizing \((1 - \kappa) \times \frac{d\tilde{S}}{dQ(p)} \) at the 10th percentile and \( d\tilde{S}/dQ(p) \) at the 90th percentile for the Atkinson and Kolm-Pollak transformation functions. After some reworking, this yields that \( \gamma = -\ln(1 - \kappa)/\ln[Q(0.9)/Q(0.1)] \) and \( \beta = -\ln(1 - \kappa)/[Q(0.9) - Q(0.1)] \) in which the appropriate percentile values of the well-being distribution and value for the welfare-neutral leak \( \kappa \) can be substituted.
sections we discuss for each well-being measure how it is operationalized and we compare the results of the two decompositions introduced in Section 2.2. First, we look at the decomposition of the level of social welfare in the mean and inequality and, second, we turn to the decomposition of the change in social welfare using the growth incidence curves.

<table>
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<tr>
<th></th>
<th>expenditures</th>
<th>equivalent income</th>
<th>vNM utility</th>
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<tr>
<td></td>
<td>( S )</td>
<td>( \mu )</td>
<td>( A )</td>
</tr>
<tr>
<td>1995</td>
<td>4863</td>
<td>4863</td>
<td>0</td>
</tr>
<tr>
<td>50%</td>
<td>4411</td>
<td>4863</td>
<td>0.093</td>
</tr>
<tr>
<td>90%</td>
<td>3505</td>
<td>4863</td>
<td>0.279</td>
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<tr>
<td>99%</td>
<td>2486</td>
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</tr>
<tr>
<td>2000</td>
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<td>0</td>
</tr>
<tr>
<td>50%</td>
<td>3699</td>
<td>4126</td>
<td>0.103</td>
</tr>
<tr>
<td>90%</td>
<td>2894</td>
<td>4126</td>
<td>0.299</td>
</tr>
<tr>
<td>99%</td>
<td>2047</td>
<td>4126</td>
<td>0.504</td>
</tr>
<tr>
<td>2005</td>
<td>5732</td>
<td>5732</td>
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</tr>
<tr>
<td>50%</td>
<td>5203</td>
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<tr>
<td>99%</td>
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Table 5: Equally-distributed equivalent \((\bar{S})\), average \((\mu)\) and inequality \((A)\) for the Atkinson social welfare function, using expenditures, equivalent incomes and vNM utilities as well-being measure

### 4.1 Expenditures

The real household expenditures are measured by adding the expenditures of all household members and applying the square root equivalence scale to capture differences in household composition. The first column of Table 5 presents the level of the equally-distributed equivalent of the Atkinson social welfare function and the first column of Table 6 shows the level of the equally-distributed equivalent of the Kolm-Pollak social welfare function. We use the inequality aversion parameters \(\gamma\) and \(\beta\) that are implied by the four values of the welfare-neutral leak \(\kappa\) (see Table 4). We see that for all parameter values of the two social welfare functions, the lowest level of social welfare is reached in 2000, in the aftermath of the Russian financial crisis. After 2000, social welfare increases again and reaches a higher level in 2005 compared to 1995.

The decomposition of social welfare in the mean and inequality (shown in the second and third column of Table 5 and 6) reveals some additional interesting findings. First, the more inequality averse the social welfare function is, the bigger the welfare loss due to inequality measured by the inequality measure \(A\) in Table 5 and \(KP\) in Table 6. Remarkably, the welfare
Table 6: Equally-distributed equivalent ($\tilde{S}$), average ($\mu$) and inequality ($KP$) for the Kolm-Pollak social welfare function, using expenditures, equivalent incomes and vNM utilities as well-being measure

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>$t$</td>
<td>$\kappa$</td>
<td>$S$</td>
<td>$\mu$</td>
<td>$KP$</td>
<td>$S$</td>
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<tr>
<td>1995</td>
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<td>0,688</td>
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<td>4863</td>
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<td>3072</td>
<td>4863</td>
<td>1791</td>
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<td>662</td>
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<td>0,688</td>
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<tr>
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<td>4126</td>
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<td>0</td>
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<td>0,680</td>
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</tr>
<tr>
<td></td>
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<td>3661</td>
<td>4126</td>
<td>465</td>
<td>928</td>
<td>1172</td>
<td>244</td>
<td>0,656</td>
<td>0,680</td>
<td>0,024</td>
</tr>
<tr>
<td></td>
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<td>3096</td>
<td>4126</td>
<td>1030</td>
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<td>502</td>
<td>0,595</td>
<td>0,680</td>
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<td>99%</td>
<td>2654</td>
<td>4126</td>
<td>1472</td>
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<td>0,511</td>
<td>0,680</td>
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<tr>
<td>2005</td>
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<td>5732</td>
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<td>0</td>
<td>1606</td>
<td>1606</td>
<td>0</td>
<td>0,704</td>
<td>0,704</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>5008</td>
<td>5732</td>
<td>724</td>
<td>1230</td>
<td>1606</td>
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<td>1591</td>
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<td>1606</td>
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<td>2256</td>
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<td>1606</td>
<td>989</td>
<td>0,552</td>
<td>0,704</td>
<td>0,153</td>
</tr>
</tbody>
</table>

loss due to inequality shows a different trend for the Atkinson and Kolm-Pollak social welfare functions over time. While it is inversely U-shaped for the Atkinson social welfare function, it is U-shaped in the case of Kolm-Pollak.

The second decomposition, relating social progress to the relative and absolute growth incidence curves, allows us to investigate this finding further. Figures 2 and 3 show the relative and absolute growth incidence curves in the top and bottom panel, respectively. Figure 2 focusses on the period between 1995 and 2000, whereas Figure 3 covers the entire period between 1995 and 2005.
Figure 2: Growth incidence and change in social welfare between 1995 and 2000 for Atkinson (top) and Kolm-Pollak (bottom) social welfare functions, using expenditures as well-being measure.
Figure 3: Growth incidence and change in social welfare between 1995 and 2005 for Atkinson (top) and Kolm-Pollak (bottom) social welfare functions, using expenditures as well-being measure.

Let us start from Figure 2. The difference between the top and bottom panel of this figure is striking. It illustrates why the social welfare loss due to inequality shows a different trend for the Atkinson and Kolm-Pollak inequality measures. In the period between 1995 and 2000, the top panel shows that the growth rate of expenditures was negative throughout the entire distribution, being more negative for the poorest individuals. The annual growth rate of expenditures reached -4.5% at the 20th percentile, while it was “only” -2.5% at the
top of the distribution. The bottom panel shows the same period, but tells a quite different story by looking at the absolute changes in expenditures (expressed in ruble), rather than their relative growth rate. The absolute reduction of expenditures during the crisis is larger for persons higher in the distribution. Both findings are not inconsistent in the presence of large initial expenditure inequalities: the loss of 2.5% of a large expenditure level at the top of the distribution turns out to be larger than 4.5% of a smaller expenditure level at the bottom. Both growth incidence curves tell a different (and complementary) story. As we have seen in Section 2.2 the relative growth incidence curve in the top panel of Figure 2 is closely related to the relative change in the Atkinson social welfare function.24 The growth rate of the Atkinson social welfare can be written as a weighted sum of the growth rates of well-being at the different percentiles with the weights being equal to the ratio of the transformed well-being at each percentile and the average transformed well-being across all percentiles. The weights at each percentile for four different $\kappa$ values are indicated in the Figures 2 and 3 in greyscale. The higher this weight, the higher the impact on the change in social welfare. The size of the weight depends on the shape of the distribution and also on the selected welfare-neutral leak $\kappa$ and the inequality aversion parameter $\gamma$ it implies (see Table 4). Remember that for the Atkinson social welfare function, the weight increases with well-being if the selected welfare-neutral leak $\kappa$ and the implied inequality aversion $\gamma$ is sufficiently low. This is exactly what we observe in the figure. For more inequality averse social welfare functions, however, the weights are larger at the bottom of the distribution, reflecting that the relative growth of social welfare is more sensitive to the well-being growth of the worse-off.

Figure 3 shows the relative and absolute growth incidence curves for the entire period between 1995 and 2005. The relative growth rates in the top panel turn out to be positive throughout the distribution, and are overall modestly pro-poor. While the 10th percentile obtains an annual growth rate of 2%, the top of the distribution reaches only 1.5%. The bottom-panel shows, however, that the absolute growth of the rich expressed in ruble remained much larger than the growth of the poor. Even with a larger annual growth rate at the bottom of the distribution, the absolute expenditure differences have increased during the considered period. The later phenomenon is picked up by the increased inequality measure $K_P$ in Table 6.

24 The relative (yearly) growth rates can be computed from the first column of Table 5 and the absolute yearly growth can be computed from the first column of Table 6.
4.2 Equivalent Incomes

We now turn to the preference-based measures of well-being. A *conditio sine qua non* to implement a preference-based well-being measure, is to know the preferences of the respondents. We follow Decancq et al. (2017), who have used a life satisfaction approach to estimate the ordinal preferences over expenditures, health, housing quality, employment status and the presence of wage arrears for the respondents in the nine waves of the RLMS-HSE between 1995 and 2005. The estimation allows for preference heterogeneity at the level of socio-demographic groups that are determined by a series of binary indicators that capture whether the respondent is living in a rural area, is young (below the age of 33), is male, and reports to have obtained higher education.

With these estimates of the ordinal preferences, the equivalent income well-being measure $y_{it}^*$ as defined by Eq. (32) in Section 3.3 can be computed for all respondents in the data set. These equivalent incomes are used as arguments of a social welfare function, similarly to the expenditures in the previous subsection. The fourth column of Table 5 and 6 shows the evolution of the (equally-distributed equivalent of the) Atkinson social welfare function and the Kolm-Pollak social welfare function, respectively. Columns five and six present the decomposition of social welfare in the mean and inequality. The parameters are set based on the leak in a welfare-neutral leaky-bucket transfer between a person on the 90th percentile of the distribution of equivalent incomes and the 10th percentile. As we have discussed in Section 3, the equivalent incomes are obtained by subtracting the willingness-to-pay to be in the reference situation for all non-income attributes, from the expenditures. Since the estimated willingness-to-pay is non-negative for all respondents, the level of social welfare using equivalent incomes as measure of individual well-being is lower than the level of social welfare with expenditures as well-being measure. Note that the evolution of equivalent income over time, as shown in Table 5 and 6, does not show the sharp drop in 2000 that is observed for expenditures.

The change in social welfare can be decomposed using the relative and absolute growth incidence curves. The top panel of Figure 4 shows that the equivalent incomes of the worse-off decrease in the initial period between 1995 and 2000 at a stunning annual rate of about 15%, but that the equivalent incomes of the 60% best-off people remains relatively stable during the financial crisis of the late nineties. The bottom panel shows, interestingly, that also for the worst-off the decrease in well-being remains modest when it is expressed in monetary values. Yet, the initial well-being levels are so low at the bottom of the distribution, that

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25 For more information on the definition of the variables, the econometric model, and the estimated coefficients, see Decancq et al. (2017). Decancq et al. (2015b) and Decancq et al. (2019) use a similar approach to estimate ordinal preferences for the RLMS-HSE.
a modest decrease in monetary value leads to a large decrease when it is expressed as a percentage. Similar to the Figures 2 and 3, the weights at each percentile for four different $\kappa$ values are indicated in the Figures 4 and 5 in greyscale. Given the low well-being values at the bottom of the distribution, the share of well-being going to the top of the distribution is large, leading to a sharp increase of the weight across the percentiles in the top panels, except for a large $\kappa$ value.

The relative and absolute growth incidence curves for the period between 1995 and 2005, shown in Figure 5, indicate social progress for almost the entire distribution. Only the bottom decile has experienced a negative relative growth rate and are worse-off in 2005 compared to 1995.
Figure 4: Growth incidence and change in social welfare between 1995 and 2000 for Atkinson (top) and Kolm-Pollak (bottom) social welfare functions, using equivalent incomes as well-being measure
Figure 5: Growth incidence and change in social welfare between 1995 and 2005 for Atkinson (top) and Kolm-Pollak (bottom) social welfare functions, using equivalent incomes as well-being measure.

4.3 von Neumann-Morgenstern utilities

To compute the second preference-based well-being measure based on von Neumann-Morgenstern (vNM) utilities, we use the “high-low” method proposed in Adler (2016) and discussed in the third chapter in this book. We start from the ordinal preferences estimated by the life satisfaction approach for the computation of the equivalent incomes. Based on the (estimated)
risk attitudes of the respondents, we select a vNM utility function that is consistent with these preferences. The cardinal well-being levels are then determined by selecting two attribute bundles where the well-being level is calibrated at the level of 0 and 1, respectively. That is, we measure well-being by the following Eq.

\[ WB = \frac{V_i(\ell_i) - V_i(\ell_i^+)}{V_i(\ell_i^{++}) - V_i(\ell_i^+)}, \]

where \( V_i \) is the vNM utility function of person \( i \), and \( \ell_i^+ \) and \( \ell_i^{++} \) are calibration points that are chosen such that \( WB = 0 \) and \( WB = 1 \) respectively. The main difference between the vNM approach and equivalent incomes is the transformation of Eq. (40) and the treatment of the cardinal information about the risk aversion of the respondents (the third chapter in this book provides a more detailed comparison between equivalent incomes and the vNM approach). Whereas cardinal information plays no role in the computation of equivalent incomes, the risk aversion determines the curvature of the vNM utility function \( V_i \) and, hence, also the well-being level of the person.\(^{26}\) To compute the well-being measures with Eq. (40), we set the attribute bundle \( \ell_i^+ \) equal to the attribute bundle that consists of the lowest value in all dimensions of well-being and \( \ell_i^{++} \) equal to the attribute bundle that consists of the 99th percentile of expenditures and the reference value for all non-income dimensions. The obtained well-being measures can then be used as argument in the Atkinson and Kolm-Pollak social welfare functions. The seventh column of Table 5 and 6 shows the evolution of social welfare measure with the (equally-distributed equivalent of the) Atkinson social welfare function and the Kolm-Pollak social welfare function, respectively. The parameters are set by means of the leaky bucket thought experiment and are given in Table 4.\(^{27}\) Column eight and nine present the decomposition of social welfare in the mean and inequality. We see that the lowest level of social welfare is reached in 2000 for all inequality aversion parameter values.

The specific cardinalization of the vNM well-being measures leads to a smaller difference between the relative and absolute growth incidence curve, as shown in Figure 6 and 7. The worse-off experience regress when the vNM approach is used to measure well-being, whereas the better-off make progress, irrespective whether an Atkinson or Kolm-Pollak social welfare function is used.

Although the broad-brushed picture about the evolution of social welfare in Russian between

\(^{26}\)The appendix describes how the risk aversion of the respondents is estimated and imputed with the RLMS-HSE.

\(^{27}\)By construction, the vNM well-being measures take values between 0 and 1. This is different from the monetary scale of expenditures and equivalent incomes. The absolute difference between the 10th and 90th percentile is smaller, as is the ratio. The corresponding \( \gamma \) and \( \beta \) are hence much larger.
Figure 6: Growth incidence and change in social welfare between 1995 and 2000 for Atkinson (top) and Kolm-Pollak (bottom) social welfare functions, using vNM utility as well-being measure
Figure 7: Growth incidence and change in social welfare between 1995 and 2005 for Atkinson (top) and Kolm-Pollak (bottom) social welfare functions, using vNM utility as well-being measure.
1995 and 2005 that the different well-being measures and social welfare functions reveal is rather similar, a more nuanced and subtle story emerges from the complementary use of the different measures and social welfare functions.

5 Conclusion

Assessing social conditions is an essential step in the evaluation of economic policies or the overall performance of economic and political systems. This is crucial for comparisons between countries as well as for analyzing social progress over time. If one accepts that such assessment should take into account that the quality of individual lives depends on more than only income and that the evaluation of a social condition involves distributional issues, the prioritarian social welfare framework is a natural candidate. In this chapter we have discussed briefly the main theoretical issues and surveyed some empirical applications. The applied literature is large and we have focused on the measurement of social progress over time rather than on international comparisons. Assessing social conditions is essentially a normative exercise and the results crucially depend on the choice of the measure of individual well-being and of the degree of inequality aversion.

Throughout the chapter we used the equally-distributed equivalent representation of social welfare and we distinguished the Atkinson and the Kolm-Pollak social welfare functions. To get a better insight in the relative importance of distributional issues, we proposed a decomposition of social welfare into the average and the inequality of individual well-being. To analyze changes in social welfare over time, we related the growth incidence curve to the prioritarian social welfare framework. All these concepts were illustrated with an empirical application on Russian data for the turbulent period between 1995 and 2005. Our application shows that the prioritarian framework is indeed a powerful tool to assess social conditions.

Appendix 1: Decomposing the change of social welfare

Using the continuous notation of Section 2.2, we define the social welfare function and its equally-distributed equivalent as follows:

\[ S_t = \int_0^1 g(Q_t(p)) \, dp. \]  
\[ \tilde{S}_t = g^{-1} \left[ \int_0^1 g(Q_t(p)) \, dp \right] = g^{-1} (S_t). \]
The absolute change in the equally-distributed equivalent $\tilde{S}_t$ over time can be written as follows:

$$\frac{d\tilde{S}_t}{dt} = \frac{dg^{-1}(S_t)}{dS_t} \times \int_0^1 \left[ \frac{dg(Q_t(p))}{dQ_t(p)} \times \frac{dQ_t(p)}{dt} \right] dp, \quad (43)$$

where the third component equals the absolute growth incidence curve $aGIC$ (Eq. (14)).

Likewise, the relative change in the equally-distributed equivalent $\tilde{S}_t$ over time can be obtained:

$$\frac{d\tilde{S}_t/\tilde{S}_t}{dt} = \frac{dg^{-1}(S_t)}{dS_t} \times \int_0^1 \left[ \frac{dg(Q_t(p)) Q_t(p)}{dQ_t(p) \tilde{S}_t} \times \frac{dQ_t(p)}{dt} \frac{1}{Q_t(p)} \right] dp. \quad (44)$$

where the third component equals the relative growth incidence curve $rGIC$ (Eq. (12)).

First, using the Atkinson transformation function

$$g_A(Q_t(p)) = \frac{Q_t(p)^{1-\gamma}}{1 - \gamma}, \quad (45)$$

we obtain that the first component of Eq. (44) equals:

$$\frac{dg_A^{-1}(S_t^{ATK})}{dS_t^{ATK}} = \left[ (1 - \gamma) S_t^{ATK} \right]^\frac{\gamma}{1 - \gamma}. \quad (46)$$

The second component of the same expression equals:

$$\frac{dg_A(Q_t(p)) Q_t(p)}{dQ_t(p) \tilde{S}_t^{ATK}} = \frac{Q_t(p)^{1-\gamma}}{[(1 - \gamma) S_t^{ATK}]^{1-\gamma}}. \quad (47)$$

Multiplying Eqs. (46) and (47) yields Eq. (13).

Second, using the Kolm-Pollak transformation function

$$g_K(Q_t(p)) = -\frac{\exp(-\beta Q_t(p))}{\beta}, \quad (48)$$

we obtain that the first component of Eq. (43) equals:

$$\frac{dg_K^{-1}(S_t^{KP})}{dS_t^{KP}} = \frac{1}{\int_0^1 \exp(-\beta Q_t(p)) dp}. \quad (49)$$
The second component of the same expression equals:

\[
\frac{dg_K(Q_t(p))}{dQ_t(p)} = \exp(-\beta Q_t(p)).
\]  

(50)

Multiplying Eqs. (49) and (50) yields Eq. (15).

Appendix 2: Imputation of the risk attitudes based on the RLMS-HSE data

To determine a level of risk aversion for each respondent in our data set, we use additional information from an ad-hoc risk module to the RLMS-HSE in 2009.\(^{28}\) This module is inspired by the work of Dohmen et al. (2011) and contains a specific question about risk attitudes: “Are you generally a person who is willing to take risks or do you try to avoid risks?” Respondents use an 11-point response scale between 0 (not at all willing to take risks) and 10 (very willing to take risks). Based on the responses to this question, the risk attitude can be predicted for each respondent in the data between 1995 and 2005 using some socio-economic variables (the coefficients are given in Table 7). These predicted risk attitudes are obviously an imperfect approximation of the risk aversion of the respondents, but the data set does not permit us to make more detailed estimates. Based on the predicted risk attitudes the respondents are classified in four equal-sized groups (called risk neutral, low risk aversion, medium risk aversion, high risk aversion, respectively). The higher the predicted risk attitude, the more curvature is assumed on the vNM utility function \(V_i\), see also Decancq and Neumann (2016).

\(^{28}\)We are grateful to Professor Vladimir Gimpelson for making this additional information available.
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Standard errors in parentheses

$^+ p < 0.10$, $^* p < 0.05$, $^{**} p < 0.01$, $^{***} p < 0.001$

Table 7: Estimation of the individual risk attitudes
References


